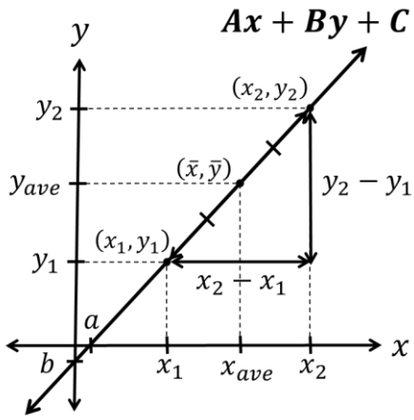
POINTS, LINES, AND CIRCLES

DISTANCE, SLOPE, AND MIDPOINT FORMULA



Distance Formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Slope of a line

$$m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

(+) slope → to the right
(-) slope → to the left

Midpoint Formula

$$\bar{x} = \frac{x_1 + x_2}{2}$$

$$\bar{y} = \frac{y_1 + y_2}{2}$$

General equation of a line

$$Ax + By + C = 0$$

STANDARD EQUATIONS OF LINES

Point-slope form

$$y - y_1 = m(x - x_1)$$

Slope-intercept form

$$y = mx + b$$

Two-point form

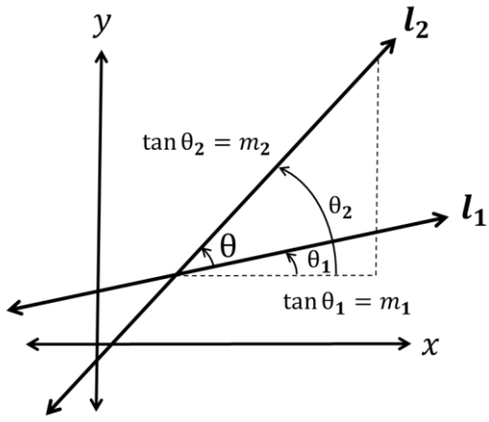
$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

ANGLE BETWEEN TWO LINES

- for same side ang inclination



$$\theta = \tan^{-1}\left(\frac{m_2 - m_1}{1 + m_1 m_2}\right)$$

or

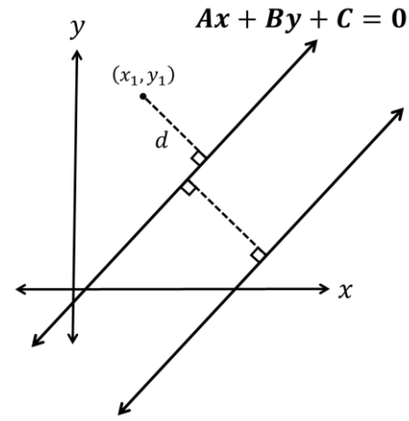
$$\theta = \tan^{-1}(m_2) - \tan^{-1}(m_1)$$

Note:

eq.1: 3x − 2y − 10 = 0
eq.2: (6x − 4y − 18 = 0) ½
3x − 2y − 9 = 0
parallel kasi same ng coefficient ang x & y.

1. Two lines are *parallel* if their slopes are equal. ($m_1 = m_2$)
2. Two lines are *perpendicular* if the product of their slopes is -1. ($m_1 m_2 = -1$)

DISTANCE FROM A POINT TO A LINE



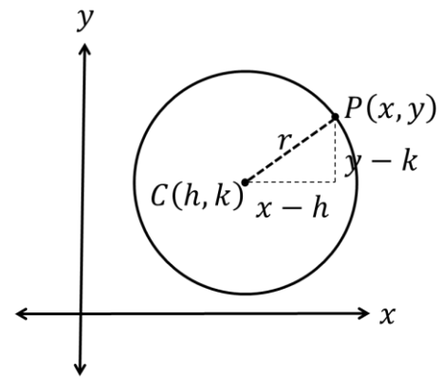
Distance from (x_1, y_1) to $Ax + By + C = 0$

$$d = \left|\frac{Ax_1 + By_1 + C}{\sqrt{A^2 + B^2}}\right|$$

Distance between parallel lines

$$d = \left|\frac{C_2 - C_1}{\sqrt{A^2 + B^2}}\right|$$

EQUATIONS OF CIRCLE



Center at $C(h, k)$

$$(x - h)^2 + (y - k)^2 = r^2$$

Center at origin

$$x^2 + y^2 = r^2$$

General form

$$x^2 + y^2 + Dx + Ey + F = 0$$

*Center(h,k): h = -D/2 k = -E/2

A locus of a point which moves at a constant distance from a fixed point called **center** and the constant distance of any point from the center is called the **radius.**

SAMPLE PROBLEMS:

Situation 1:
For problems 1 - 8, refer here. Given the triangle with vertices at A(1,4), B(9,6), and C(7,2). Find the:

1. equation of the line through side AB
   a. $x + 4y + 15 = 0$ c. $x + 4y - 15 = 0$
   b. $x - 4y + 15 = 0$ d. $x - 4y - 15 = 0$
2. distance from C to side AB
   a. 3.4 c. 4.0
   b. 4.3 d. 3.7
3. equation of the line through (0,-3) and parallel to side AB
   a. $x + 4y + 12 = 0$ c. $x + 4y - 12 = 0$
   b. $x - 4y + 12 = 0$ d. $x - 4y - 12 = 0$
4. equation of the perpendicular bisector of side BC
   a. $x - y - 8 = 0$ c. $x + 2y - 16 = 0$
   b. $x - 2y + 8 = 0$ d. $x + y + 16 = 0$
5. angle C of the triangle
   a. 98.1° c. 81.9°
   b. 89.8° d. 91.1°
6. terminal point if side AC is extended three times its own length from C
   c. (24,-6) c. (24,-5)
   d. (25,-4) d. (25,-3)
7. area of the triangle
   a. 15 sq. units c. 13 sq. units
   b. 12 sq. units d. 14 sq. units

CE BOARD NOV. 2021

8. In a city, streets run north and south, and avenues run east and west. Streets and avenues are 850 feet apart. The city plans to construct a straight freeway from the intersection of 25th street and 8th avenue to the intersection of 115th street and 64th avenue. How long will the freeway be?
   A. 13 miles C. 15 miles
   B. 11 miles D. 17 miles

Euler's line – a line that contains orthocenter, centroid and circumcenter.

9. The equation of the Euler's line of triangle ABC is 51x - 71y - 122 = 0. The vertices of the triangle are: A(-2, 3), B(1, -7) and C(x, 1). Locate the triangle's orthocenter.

• if the line passes through the point, it means the coordinates

$(3.63, 0.89)$, $(\frac{98}{7}, \frac{2}{9})$

• evaluating the point of intersection involves

Situation 2.
For problems 10-13, refer here. A circle has the equation $x^2 + y^2 - 4x + 6y - 12 = 0$. Find the:

10. center and radius of the circle
    a. (2,-3); r = 25 c. (2,-3); r = 5
    b. (-2,3); r = 5 d. (-2,3); r = 25
11. farthest distance from the point (5,6) to the circle
    a. 14.2 units c. 15.4 units
    b. 14.5 units d. 15.2 units
12. nearest distance from the point (5,6) to the circle
    a. 5.4 units c. 4.2 units
    b. 5.2 units d. 4.5 units
13. tangent distance from the point (5,6) to the circle
    a. 8.1 units c. 8.6 units
    b. 9.6 units d. 9.1 units

14. Find the equation of the circle having its center on the line 4x - y = 7 and passing through the points (-2, 4) and (5, 5).

$(h+2)^2 + (k-4)^2 = (h-5)^2 + (k-5)^2$ $(x-2)^2 + (y-1)^2 = 25$

$4h - k = 7$ $h = 2$ $r = 5$
$k = 4h - 7$ $k = 1$

15. Find an equation(s) of the circle(s) tangent to both axes and containing the point (-8, -1).

$r_1 = 5$; $(-8, -5)$; $(x+5)^2 + (y+5)^2 = 25$
$r_2 = 13$; $(-13, -13)$; $(x+13)^2 + (y+13)^2 = 169$

Problems for Practice:

1. The lines $2x + ay + 2b = 0$ and $ax - y - b = 1$ intersect at the point (-1,3). What is $2a + b$?

2. Find the equation of the line passing through the point A(2, -3) and perpendicular to the line having the parametric equations:
   $5x = 3t + 4$ and $3y = 4t - 6$

3. Determine the equations of the lines (two answers) passing through (-2, 4) and forming with the axes a triangle having an area of 9 square units.

4. Find the farthest distance from the point (12, 2) to the circle $x^2 + y^2 + 6x - 16y + 24 = 0$.

5. The two points on the line $2x + 3y + 4 = 0$ which are at distance 2 from the line $3x + 4y - 6 = 0$.

6. A line has an equation of $x + 5y + 5 = 0$. Find the equation of the line through (3, 1) that makes an angle of 45° clockwise from the line that is perpendicular to the line $x + 5y + 5 = 0$ at that point.

7. Find the equation of the circle circumscribing the triangle with vertices at A(-1, -4), B(3, -2) and C(5, 2).

8. Determine the length of the tangent to the circle $x^2 + y^2 - 4x - 5 = 0$ from (8, -2).

9. Find the equation of the circle a diameter of which is the line segment connecting the centers of the following circles:
   $x^2 + y^2 + 2x + 2y - 7 = 0$ and
   $x^2 + y^2 - 4x + 8y - 5 = 0$

10. What is the radius of a circle with the equation
    $2x^2 + 2y^2 - 3x + 4y - 1 = 0$?

11. Find the coordinates of the point which is 3/8 of the way from the point A(-1, 3) to the point B(4, -2).

Answers:
1. *6 units*
2. $9x + 20y + 42 = 0$
3. $x + 2y = 6$ *and* $8x + y = -12$
4. *23.16 units*
5. *(64, - 44) and (4, -4)*
6. $2x - 3y = 3$
7. $(x + 2)^2 + (y - 3)^2 = 50$
8. *5.57 units*
9. $(x - 1/2)^2 + (y + 5/2)^2 = 9/2$
10. $\sqrt{33}/4$
11. *(7/8, 9/8)*

SAMPLE PROBLEMS:

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   a. 15 sq. units c. 13 sq. units
   b. 12 sq. units d. 14 sq. units
7. point of intersection of the medians
   a. (14/3, 3) c. (17/3, 4)
   b. (15/3, 4) d. (16/3, 3)
8. The equation of the Euler's line of triangle ABC is 51x – 71y – 122 = 0. The vertices of the triangle are: A(-2, 3), B(1, -7) and C(x, 1). Locate the triangle's orthocenter.

Situation 2. For problems 9-13, refer here. A circle has the equation $x^2 + y^2 - 4x + 6y - 12 = 0$. Find the:

9. center of the circle
   a. (-2,3) c. (2,-3)
   b. (-3,2) d. (3,-2)
10. area of the circle
   a. 78.5 sq. units c. 75.8 sq. units
   b. 87.5 sq. units d. 85.7 sq. units
11. farthest distance from the point (5,6) to the circle
   a. 14.2 units c. 15.4 units
   b. 14.5 units d. 15.2 units
12. nearest distance from the point (5,6) to the circle
   a. 5.4 units c. 4.2 units
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15. Find an equation(s) of the circle(s) tangent to both axes and containing the point (-8, -1).

Problems for Practice:

1. Find the coordinates of the point which is 3/8 of the way from the point A(-1, 3) to the point B(4, -2).

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7. Find the equation of the circle circumscribing the triangle with vertices at A(-1, -4), B(3, -2) and C(5, 2).

8. Determine the length of the tangent to the circle $x^2 + y^2 - 4x - 5 = 0$ from (8, -2).

9. Find the equation of the circle a diameter of which is the line segment connecting the centers of the following circles:
   $x^2 + y^2 + 2x + 2y - 7 = 0$ and
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10. What is the radius of a circle with the equation

$$2x^2 + 2y^2 - 3x + 4y - 1 = 0?$$

11. The lines $2x + ay + 2b = 0$ and $ax - y - b = 1$ intersect at the point (-1,3). What is $2a + b$?

Answers:

1. (7/8, 9/8)
2. 9x + 20y + 42 = 0
3. x + 2y = 6 and 8x + y = -12
4. 23.16 units
5. (64, - 44) and (4, -4)
6. 2x – 3y = 3
7. $(x + 2)^2 + (y - 3)^2 = 50$
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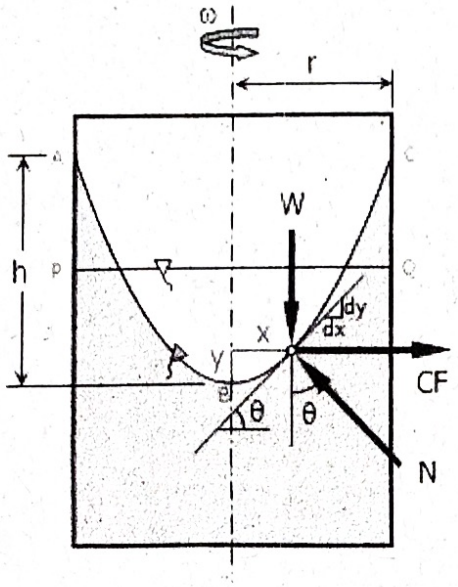
9. $(x - 1/2)^2 + (y + 5/2)^2 = 9/2$
10. $\sqrt{33}/4$
11. 6 units

ROTATING VESSEL



$$\tan\theta = \frac{\omega^2 x}{g}$$

Where tan θ is the slope at the surface of paraboloid at any distance x from the axis of rotation.

$$y = \frac{\omega^2 x^2}{2g}$$

By squared-property of parabola,

$$\frac{r^2}{h} = \frac{x^2}{y}$$

Important conversion factor

$$1\ \text{rpm} = \frac{1}{30}\pi\ \text{rad/sec}$$

1. Determine the rpm required for an open 100-cm Ø cylindrical tank 250 cm high, containing water 150 cm deep, so that:
   a. Water could just reach the rim of the tank.
   b. The depth of water at center is zero.
   c. There is no water at the bottom within 20 cm from the axis of rotation.

2. An open cylindrical tank 800 mm in diameter and 2000 mm high contains water to a depth of 1.6 m. It is rotated about its vertical axis at 13 rad/s. How many liters of water are spilled?

3. A 1-m Ø closed cylindrical vessel with vertical axis is 3 m high. It is filled with water with a pressure of 290 kPa at the top. The wall is 4 mm thick. If the vessel is rotated at 360 rpm,
   a. Determine the total pressure against the top.
   b. Determine the maximum intensity of hoop tension.
   c. Determine the force of water against the wall.

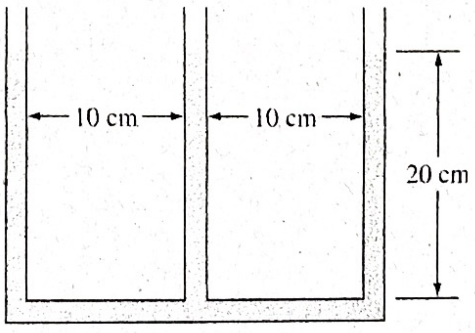
4. A closed cylinder 1.0 m Ø and 2.0 m high contains water 1.5 m deep, the remaining air space being under a pressure of 120 kPa.
   a. Compute the pressure at the center of the bottom if ω = 12 rad/s.
   b. Compute the pressure at the circumference of the bottom.
   c. What ω will just zero the depth of water at the center.
   d. How much area of the bottom is not covered with water if ω = 20 rad/s?

5. An open 1 – m diameter tank contains water at a depth of 0.7 m when at rest. As the tank is rotated about its vertical axis, the center of the fluid surface is depressed. At what angular velocity will the bottom of the tank first be exposed? No water is spilled from the tank. **Ans: 10.5 rad/s**

6. An open, 0.60-m-diameter tank contains water to a depth of 1.0 m when at rest. If the tank is rotated about its vertical axis with an angular velocity of 180 rev/min, what is the minimum height of the tank walls to prevent water from spilling over the sides? **Ans: 1.815 m**

7. A closed, 0.4-m-diameter cylindrical tank is completely filled with oil (SG = 0.9) and rotates about its vertical longitudinal axis with an angular velocity of 40 rad/s. Determine the difference in pressure just under the vessel cover between a point on the circumference and a point on the axis.
   Ans: 28.8 kPa

8. The three-legged manometer shown is filled with water to a depth of 20 cm. All tubes are long and have equal small diameters. If the system spins at angular velocity of 120 rpm about the central tube,
   a. Find the height in cm in the central tube. [Hint: The central tube must supply water to both the outer legs.]
      Ans: 14.7
   b. Find the height in cm in the outer tube.
      Ans: 22.7



9. An open cylindrical tank, 1.0 m in diameter and 2.0 m high has 1.5 m water,
   a. What constant rpm can be given to the cylinder without causing any water to spill?
      Ans. 84.6
   b. What is the pressure at the center of the bottom of the tank when it is rotated at 57.3 rpm? **Ans. 12.5 kPa**
   c. What is the pressure at the circumference of the bottom of the tank when it is rotated at 57.3 rpm? **Ans. 17 kPa**